November 2, 2022

Week 10 Notes:
Intrinsic Metavocabularies for Reason Relations

I have to start with the confession that I am going off-syllabus this week.
And, as these things do, the divergence is liable only to ramify in coming weeks.
There is just too much that should be said about the relations between the various claims, concepts, and constructions that have been put on the table in the seminar so far for me to jump into the further considerations articulated in the original plan.
We'll get back to that material eventually, even if only in abbreviated form.

I apologize for only getting the changes up on the website this morning—including a new, different reading (which I obviously do not assume anyone has had a chance to look at).
It there just because it is the reading most relevant to what I am actually going to talk about.
Let's skip to the old Week 11 for next week's readings.

Plan:

1. Bimodal conceptual realism and Ulf's isomorphism.
2. Implication-space semantics and Bob's isomorphism.
3. Truth-value model theory vs. Inferential Entailment Roles

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I. Conceptual Realism

McDowell in *Mind and World*: "The conceptual has no outer boundary."
When we think about how things are, we should not understand that as *crossing a boundary* between a subjective activity (thinking) that is conceptually structured or articulated and an objective reality that is not.
Only by giving up that picture (of the conceptual as bounded, as confined to the mind) can we understand how the visible *fact that* the apple is red can not only *cause* me to *believe that* the apple is red, but *justify* that belief, give me a *reason* for that belief.
If the possibility of knowledge is to be intelligible, we must understand how facts serve as reasons for belief in the sense that they can entitle us to doxastic commitments.

Wittgenstein says: "When we say, and mean, that such-and-such is the case, we—and our meaning—do not stop anywhere short of the fact; but we mean: this—is—so." [*PI* §95.]

But how can this be so?

McD: Facts and beliefs are both in conceptual form.
They must be, else we could not understand how the world objectively is, the facts, as not just *causing* our beliefs, but *justifying* them: giving us *reasons* for them.

What does that mean?
Both the *fact that* the apple is red and my *belief that* the apple is red can be stated, expressed by declarative sentences and sentential clauses: the fact that *p*, the belief that *p*.
To be statable, thinkable, claimable, expressible by declarative sentences—what we can accept or reject, manifesting practical attitudes of acceptance/rejection—is to be in conceptual form.

We can fill in this thought a little bit more by adding one of Wittgenstein's earlier, *Tractarian* thoughts: "The world is everything that is the case. It is the totality of facts, not of things."

And **Frege**: "A fact is a thought that is true."
Here by "a thought" he does not mean an *act* of think*ing*, but a think*able*—that is, what I have been calling a *claimable*.
Frege is here saying just what the *later* Wittgenstein was saying: the way things objectively are is thinkable, sayable, claimable.

I will call this line of thought, articulated by Frege, Wittgenstein, and McDowell, "**conceptual realism**."

I think one of the deepest metaphysical issues is how to think about the relation between **the sense in which the world is a totality of *facts*** and **the sense in which it consists of *objects***—and, we must immediately add, their properties and relations, which is to say facts *about* them.
The sense in which facts are *about* objects is supposed to owe nothing to our linguistic activities—after all, there were facts about objects long before there were thinkers or claimers, and there *would* have been facts about objects, objects would have had properties and stood in relations to one another, even if there never *were* thinkers or claimers.
(What in *ASoT* I call "objective idealism" claims that although all those subjunctive conditionals are true, nonetheless we cannot *understand* the sense in which facts are facts *about* objects, apart from understanding the use of declarative sentences and singular terms—and, so, predicates.)

But all this concerns not only what *sentences* express, but also *sub*-sentential expressions, and that goes beyond our topic in this course. (Sentences are complicated enough on their own.)
[Maybe gesture forward here to—or look back from there to here—discussion of Lewis's "General Semantics" treatment of adverbs in terms of ((T/S)/S) / ((T/S)/S) s.]

McDowell is content to mean by "conceptual" what Wittgenstein and Frege meant: what is expressible in declarative sentences.

(Maybe there are facts that *we*, with *our* language, cannot express—or cannot *yet* express? But on this line they are *and must be understood to be* the *kind* of thing that *can* be expressed in sentences.)

II. Reason Relations as Articulating the Conceptual as Such

But we have put ourselves in a position to be much more specific than this about the conceptual. For **we have considered a variety of metavocabularies for talking about claimables: pragmatic, logical, and two kinds of semantic metavocabularies (truthmaker and implication-space).**
Indeed, to talk about the conceptual shape Wittgenstein, Frege, and McDowell claim is shared by the objective world of facts and the subjective world of claimables-thinkables is to talk about the relations among what is expressed in *pragmatic* metavocabularies and what is expressed in ordinary empirical descriptive vocabularies (including natural-scientific ones).

And what I have urged that we focus on in thinking about the relations among these various vocabularies, as underlined and emphasized by the relations among the different metavocabularies, is what I have called *reason relations*, of implication or consequence and material incompatibility or incoherence.

- We have seen how such relations can be defined in a *two-sorted deontic normative bilateral pragmatic metavocabulary*, which codifies relations of being a reason for and being a reason against as they figure in rational defenses and challenges of claimings in terms of commitments and preclusions of entitlements to accept or reject.
- And we have seen how those same reason relations of implication and incompatibility can be made explicit in the form of claimables expressed by declarative sentences formed using distinctively *logical* vocabulary of conditionals and negation.
- We saw further how reason relations of implication and incompatibility among declarative sentences can be determined *semantically* by semantic assignments of pairs of sets of mereologically structured worldly *states* as their truthmakers and falsemakers.

The constellation of these metavocabularies, in relation to one another, articulates, gives shape and substance to, a positive suggestion for understanding the claimables expressed by the use of declarative sentences, and so to the *conceptual* as such.
To be in conceptual shape or to have conceptual content is to stand in reason relations of consequence and incompatibility to other such items.
(Already in this formulation we see the alternatives of thinking in terms of conceptual *form* and thinking in terms of conceptual *content*, which will shape Hlobil's Aristotelian hylomorphic metavocabulary and Brandom's Hegelian hylomorphic metavocabulary.
But that is a point for later on.)

Understanding conceptuality in terms of standing in reason relations offers a substantial metaconceptual advance over the Frege-Wittgenstein-McDowell characterization in terms of what is expressed by declarative sentences.

The advance consists in what we have learned, what we have become able to say, about the relations between reason relations and the various kinds of discursive metavocabulary we have canvassed.

What work can be done by the idea of **understanding the conceptual in terms of reason relations**?
Specifically, how can it help us understand *conceptual realism*?

III. Deontic Normative and Alethic Modal Vocabularies for Specifying Relations of Consequence and Incompatibility

Consider the very first distinction that articulates the idea of reason relations: the distinction between two generic *kinds* or *dimensions* of reason relations, implication or consequence and material incompatibility or incoherence.
This overarching distinction of two flavors of reason relation first showed up for us in *deontic normative* form in pragmatic metavocabulary for talking about what subjects *do* in engaging in discursive practices.
Here, by *explicitly* precluding entitlement to *reject* or *deny* some claimables, we saw how commitments to *accept* can *implicitly* commit one to accept some further claimables.
This is a relation of *consequence* or implication understood in *deontic normative* terms, relating practitioners' normative social statuses of commitment and entitlement.
And this specification of *implication* is paired with one of *incompatibility*, also specified in a deontic normative vocabulary, where commitment to accept some claims precludes entitlement to accept others (and, as we saw, *vice versa*).

This is how reason relations show up on the side of the *discursive activity* of linguistic practitioners who perform speech acts of asserting and denying, adopt practical attitudes of accepting and rejecting doxastic commitments, and rationally defend and challenge entitlement to those commitments by giving reasons for and against them.

But relations of consequence and incompatibility show up in a different guise, are specifiable in a different sort of vocabulary, in the objective world in which discursive practitioners live, and move, and have their being—the world they act in and seek knowledge of.
The fact that this coin is copper has as a *necessary consequence* the fact that it will melt at 1084° C., and will conduct electricity.
And that fact is *incompatible* with the coin's remaining solid at 2000° C. and with its being an electrical insulator, in that it is *impossible* that it be copper and solid at 2000° or that the flow of electrical current through it is substantially impeded.
Talk of objective consequences in terms of "necessity" and of objective incompatibilities in terms of "impossibility" is talk in an *alethic modal* vocabulary, underwritten by laws of nature.

So **we see the two kinds of relations, of consequence and of incompatibility, showing up in two different guises, as specifiable in two different vocabularies**.

- On the *subjective* side of *thought*, of the discursive activity of claiming and defending and challenging claims, consequence and incompatibility concern normative statuses of commitment and entitlement, specified in a *deontic normative* vocabulary.
- On the *objective* side of *fact*, consequence and incompatibility concern the necessity and impossibility, of concomitance of ways the world could be, specified in an *alethic modal* vocabulary whose distinctive early modern use was in formulating laws of nature.

Speech acts of assertion and denial, and the practical attitudes of acceptance and rejection that they manifest, are conceptually contentful because they stand to other such possible speech acts and attitudes in relations of consequence and incompatibility, understood in deontic normative terms of obligation and prohibition. These are *reason relations* because they articulate what commitments are reasons for and against what other commitments. They are implicit in practices of rationally *defending* and *challenging* assertions and denials.

Possible ways the world could objectively be are conceptually contentful because they stand to one another in attitudes of consequence and incompatibility, understood in alethic modal terms of necessity and impossibility.

Conceptual contentfulness is playing a role in relations of consequence and incompatibility.

Because these reason relations can come in two flavors, deontic and alethic, thoughts and facts are both intelligible as conceptually articulated.

This view is ***bimodal*** **conceptual realism**.

The two "modes" are deontic normative, on the *subjective* side of discursive activity, and alethic modal, on the *objective* side of how things are.

These are literally two forms of *modal* vocabulary: deontic and alethic modalities.

And the overall *telos* of claiming and acting (knowing and doing) is that the belief and the fact (in taking-true) or the intention and the fact (in making-true) should play the *same* role in relations of consequence and incompatibility, *modulo* the difference between deontic normative and alethic modal specifications of those relations.

It is because that can be so that, as Wittgenstein says in the quote with which I began:

"When we say, and mean, that such-and-such is the case, we—and our meaning—do not stop anywhere short of the fact; but we mean: this—is—so."

It is what is behind Frege's definition of a *fact* as a true *thought*.

And it is how I think we should read McDowell's claim that the conceptual has no outer boundary.

From this point of view, truthmaker semantics (like its Tarskian model-theoretic and possible worlds predecessors) shows up as a semantic metavocabulary for codifying this isomorphism of reason relations.
That is what is to govern assignments of sets of modally articulated (possible/impossible) states as truthmakers and falsemakers of sentences as their semantic intepretants.
The point is to determine the right (isomorphic) relations of consequence and incompatibility among the sentences, as determined by the modal structure of the universe on which the semantics is defined.

[a) Setting up for later weeks: Epistemic tracking and normative governance as the two relations between contents specified in deontic and alethic vocabularies. Epistemic tracking of fact by thought is itself specified in an alethic modal vocabulary of subjunctive conditionals, and normative governance of thoughts by facts is specified in a deontic normative vocabulary of authority and responsibility (translatable into talk of commitments and entitlements).
b)Also for later weeks:
Q: Why alethic modal vocabulary? (We have explained why pragmatic metavocabulary should be deontic-normative.)
A: Its relation to OED base vocabulary.
Kant-Sellars thesis about modality, relating alethic modal vocabulary (as *logical*, Kantian-*categorial* vocabulary) to OED vocabulary.]

IV. Hylomorphic Meta-Metavocabularies

Two versions of the *content* that is common, that read the hylomorphic metaphor of form and matter differently:

a) One *content* specified in two modal *metavocabularies*: normative pragmatic and alethic semantic. (Bob's Hegel.)
b) One *form* for two *matters*: mind and world. (Ulf's Aristotle.)

The difference lies in how we construe the *role w/res to relations of consequence and incompatibility* that is *common* to what is expressed in alethic modal (truthmaker) representational semantic metavocabulary and what is expressed in deontic normative bilateral pragmatic metavocabulary.
I think of it as *conceptual content,* and Ulf thinks of it as *rational form*.
We both think of that common *rational-conceptual* element as showing up two *guises*.
I call the guises different "forms" of that "content."
Ulf calls the guises different "matters" that can exhibit that same "rational form."

We can agree that we are articulating "**bimodal *hylomorphic* conceptual realism**."
We read the conceptual metaphor of hylomorphism differently.

But what is most important here is to realize that **what is at stake is our understanding of *intentionality*** in general: the relations between *mind* and *world*, the activities of knowers and agents and the world known and acted in and on.

Bimodal conceptual realism gives us a model of the shared *rational* or *conceptual* structure in virtue of which discursive practitioners—participants in games of making claims and giving and asking for reasons for them—can say and think how things are with the world consisting of the mereologically structured states that make what they say true or false.

As given definite mathematical shape by the isomorphism Ulf describes, bimodal conceptual realism is not so much a correspondence theory of truth as **a mathematically elaborated account of the conceptual structure that makes thinking that things are thus-and-so so possible, and intelligible to us as such**.

Again, we are considering an account of what it means and how it is possible for thought to be *about* the world, for our subjective claims and commitments to answer for their correctness to how the world objectively is.
The sort of account we are developing might not be right.
But understanding intentionality in terms of conceptual contents or rational forms as roles in relations of consequence and incompatibility to be able to show up in two forms: one specifiable

in deontic normative vocabulary and the other in alethic modal vocabulary is one that can be worked out in mathematical detail.

I find this an exciting prospect.

The stakes are high.

V. Implication-Space Semantics as the *Intrinsic* Metavocabulary of Reason Relations

Ulf demonstrated and constructs an isomorphism between the consequence relations (and reason relations generally)

- that are specified in a bilateral deontic normative pragmatic metavocabulary (whether deontically single-sorted, as in Restall-Ripley formulation, or deontically double-sorted as in the Brandom-Simonelli formulation) and
- those specified in a representational model-theoretic truthmaker semantic metavocabulary—provided we use the right semantic definitions of implication (and, as we'll see, incompatibility).

This result focuses our attention on the reason relations that are common to these two settings: the *pragmatic* and the *semantic*.
We understand those settings in terms of the different kinds of rational metavocabulary appropriate to each.

Seeing that they can show up in these two guises highlights the task of characterizing the conceptual contents or rational forms that declarative sentences express in virtue of the roles they play in reason relations, regardless of whether those reason relations are themselves specified in pragmatic or semantic terms.

Ulf and I agree that Dan's **implication-space metavocabulary does that**.
It is an *abstract* specification of *conceptual content*—construed, if we like, as *rational form*.
What it abstracts *from* is

- the *subjective* activities by which the practitioners who deploy or use a vocabulary to *take* claimables to be true and
- the worldly states that *make* those claimables true.

So a big part of the bimodal hylomorphic conceptual realist (BMHCR) philosophical interpretation of the isomorphism *at the level of reason relations* between bilateral pragmatics and truthmaker semantics *is* the fact that the implication space CRS captures (**expresses, makes explicit**) what is *common* to the bilateral pragmatics and the truthmaker semantics.

Implication-space CRS is a *meta*-metavocabulary, for *roles* w/res to reason relations, regardless of whether those reason relations are understood in *pragmatic* terms (i.e. in pragmatic MVs) or in (model-theoretic) *semantic* terms (i.e. in semantic MVs).

In fact, I want to claim something even stronger.

I think **the implication-space metavocabulary is something like the *native* metavocabulary of conceptual roles**, construed as roles with respect to reason relations.
It is, as I want to put it, **the *intrinsic* metavocabulary of the conceptual as such**.

Let us review what we have learned.

We now have *two* metavocabularies for codifying *arbitrary*, structurally open reason relations, the *logical* metalanguage of NM-MS and the *conceptual role* metalanguage of implication spaces.

1. **The logical metavocabulary we claimed was in principle and ideally a *universal* and *comprehensive* rational metavocabulary**.

By that we meant that *any* base vocabulary can be systematically elaborated into a logically extended vocabulary with the expressive power to make explicit the *reason relations* (that is why it is a *rational* metavocabulary) of the *original* vocabulary and at the same time the reason relations of the *logically extended* vocabulary.

For these purposes, we are thinking of a vocabulary as a pair consisting of a lexicon or set of sentences, and a set of reason relations defined on that lexicon.

Those reason relations, in turn, can be represented by a set of pairs of sets of sentences of the lexicon, $<\Gamma,\Delta>$, where $<\Gamma,\Delta>$ being in the set means that Γ implies Δ, and if Δ is empty, that Γ is incoherent—so that any two of its subsets whose union is Γ are incompatible with one another.

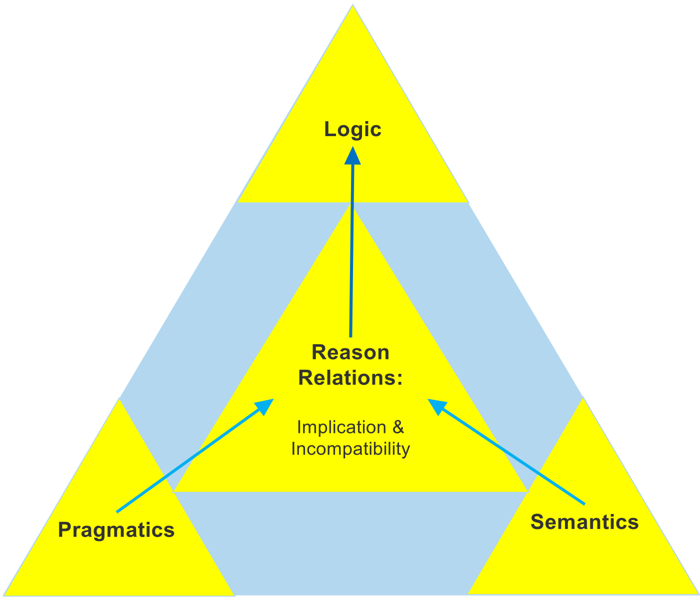
We showed that the rules of NM-MS can be applied to *any* such vocabulary, regardless of whether or not it satisfies *any* structural conditions—paradigmatically *monotonicity*, which stipulates that if Γ implies Δ then so do all of its supersets, and *transitivity* that ensures that implications of implications are already implied.
(I should note for the record that if we want to apply NM-MS without a commitment to the structural principles that Gentzen called "Contraction" and "Expansion"—which together stipulate that it does not matter how many times a premise occurs in a premise-set—then we need to formulate everything in ***multi*sets rather than just *sets***. But NM-MS works just fine with multisets, so I have refrained from discussing this nicety.)

2. **The implication-space conceptual role metavocabulary is *also* universal across arbitrary base vocabularies**.

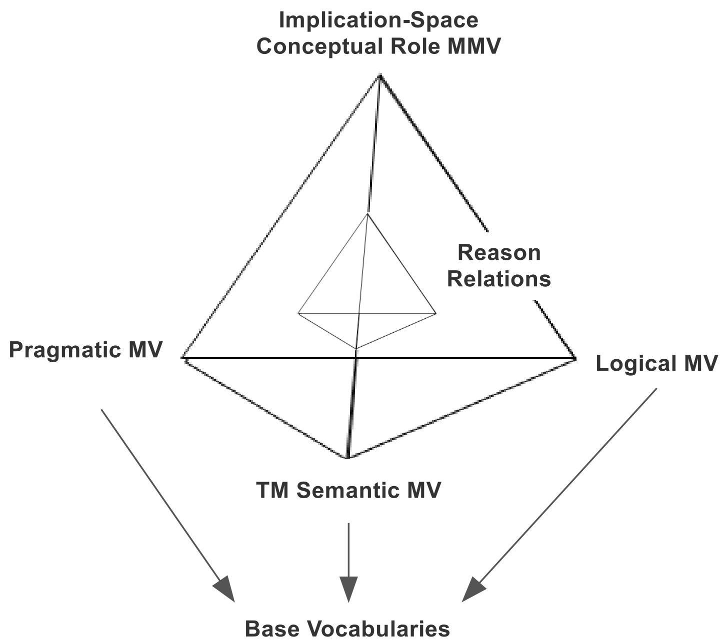
It is also, in a distinctive sense, a universal rational *meta*metavocabulary.
For it has the expressive power to codify the conceptual roles of sentences that are conferred on them by standing in reason relations as specified in pragmatic, semantic, and logical metavocabularies.

Slogan: "**One rational (meta)metavocabulary to rule them all!**"

If we think about our original articulated triangular mandala of metavocabularies that are *rational* metavocabularies in the sense that they are vocabularies for specifying the reason relations of other vocabularies:



Then we should **think of the implication-space conceptual role meta-metavocabulary as sitting above it, in a third dimension, and of the whole structure as a triangular tetrahedron with the original triangle as its base**:



I then construe the reason relations that are at the center of this metaconceptual structure as an inner pyramid, whose vertices stand in relations to the vertices of the rational metavocabularies of the outer pyramid in the distinctive expressive relations I have been rehearsing.
[Going wire-frame here has forced me to go monochromatic as well, given my poor rendering skills. Sorry about that.]

I want to contrast: *intrinsic, conceptual role* semantics and *extrinsic, representational* or *metaphysical* semantics.

What intrinsic semantics is intrinsic *to* is the *vocabulary* for which it offers a semantics.

Earlier I recalled that vocabularies can be specified as ordered pairs $\langle L, I\rangle$, where $I \subseteq \mathcal{P}(L) x \mathcal{P}(L)$.

Given *only* a *vocabulary* in this technical sense, implication-space semantics yields a *semantics* for that vocabulary. (A distant model might be Lindenbaum algebras in the setting of matrix or multivalued semantics, a topic I return to briefly later.)

Further, if we use the rules of a sequent calculus to *extend* that base vocabulary to a *logically extended* vocabulary *elaborated from* it (both its lexicon and its reason relations determined by and computable from the lexicon and reason relations of the base vocabulary), then the *very same* intrinsic conceptual role semantics that characterizes the base vocabulary can be mechanically extended (in a sound and complete way) to an intrinsic conceptual role semantics for the logically extended language.
(Fine can do the same thing with his truthmaker semantics.)
This means that implication-space conceptual role semantics is ***comprehensive*** in a sense analogous to (though different in detail from) the sense in which NM-MS is comprehensive.
For its rendering of conceptual roles extends smoothly to the conceptual roles expressed by all the sentences of the logically extended language.
That is what Dan's soundness and completeness theorems for NM-MS on the implication-space semantics shows.

The *intrinsicness* ("intrinsicality"?) of the semantics consists in its needing *nothing* else in addition to the base vocabulary to determine the whole semantics:

i) The *universe* is the set $\mathcal{P}(L) x \mathcal{P}(L)$, thought of as *candidate implications* $\langle \Gamma,\Delta\rangle$. It is determined entirely by the lexicon L of the base vocabulary.

ii) The *mereological* element of *structure* on that universe is the *commutative monoid* of *adjunction*, which is wholly definable set-theoretically from the structure of the elements of the implication space. $X \uplus Y = Z$, where $X=\langle X_1,X_2\rangle$, $Y=\langle Y_1,Y_2\rangle$ and $Z=\langle Z_1,Z_2\rangle$ iff $X_1 \cup Y_1 = Z_1$ and $X_2 \cup Y_2 = Z_2$.

iii) Further *modal* structure on the universe is the *distinguished subset* $\mathbf{I} \subseteq \mathcal{P}(L) x \mathcal{P}(L)$ of elements $<\Gamma,\Delta>$ where $\Gamma|\sim\Delta$, the *good implications* (including incoherent sets, so incompatibilities) of the base vocabulary.
There might be constraints on **I**, such as that all candidate implications of the form $<\Gamma \cup \{A\}, \Delta \cup \{A\}>$ are elements of **I**. (That is CO.) But this is determined wholly by the base vocabulary.

iv) The *space of semantic interpretants* of sentences and sets of sentences (to be assigned by the *v* function in (v)) is then the set of all sets of pairs of sets of sentences: $S = \mathcal{P}(\mathcal{P}(L) x \mathcal{P}(L))$.

v) The *interpretation function v* assigns $<X,Y> \in v(<\Gamma,\Delta>$ iff $<\Gamma \cup X, \Delta \cup Y> \in \mathbf{I}$.

This is the *range of subjunctive robustness* of an implication: the additions of premises and conclusions that would *keep* it good or *make* it good.

vi) In terms of these semantic interpretations of (candidate) *implications*, we can then assign *inferential roles* to individual sentences. Each sentence is assigned the ordered pair of (the *v*-closures of) the *v*-set of $<A,\varnothing>$, A's *premissory role*, and the *v*-set of $<\varnothing,A>$, A's *conclusory* role.

vii) We can now define not only reason relations of implication and incompatibility for the original vocabulary, but also for the logically extended vocabulary definable (elaborated) from that base vocabulary.

(Semantic Entailment). We say that A semantically entails B relative to a model M if the closure of the combination of A (as premise) and B (as conclusion) consists of only good implications:

$$A \models_M B \text{ iff } (([A]_P)^v \cup ([B]_C)^v)^{vv} \subseteq \mathbf{I}_M.$$

The closure of the adjunction of A as a premise with B as a conclusion consists only of good implications.

Q: Why not just say $A \models_M B$ iff $A|\sim B \in \mathbf{I}_M$?
A: Because the *v*-sets of logically compound sentences are defined entirely in terms of **I** determined for the base. **I** does not contain any implications involving the new logical vocabulary.
But it will follow that *if* $<A,B> \in \mathbf{I}_M$, *then* $A \models_M B$.

viii) And we can define various *metainferential* roles, beginning with *premissory* and *conclusory* roles, in addition to roles w/res to the *internal* relation of implication. (More on this later.)

By contrast, in the *extrinsic* truthmaker representational semantics:

i) The *universe* is a set of *states*, that must be **stipulated**, in some *metaphysical* vocabulary quite distinct from the base vocabulary for which a semantics is to be provided.

ii) The *mereological* element of *structure* on that universe is a *commutative monoid* of *fusion*, which must also be **stipulated**, in some *metaphysical* metavocabulary quite distinct from the base vocabulary for which a semantics is to be provided.

iii) Further *modal* structure is provided by a distinguished subset of the universe, the *possible* states, which must also be **stipulated**, in some *metaphysical* metavocabulary quite distinct from the base vocabulary for which a semantics is to be provided.

iv) The *space of semantic intepretants* of sentences (and sets of sentences), to be assigned to sentences by the interpretation function in (v), then is the set of pairs of sets of states. This can be *defined* from (i) and (ii).

v) The *interpretation function* that assigns each sentence (or set of sentences) a pair of sets of states, as its (exact) *truthmakers* and (exact) *falsemakers*. This must be **stipulated**, in some (semantic, metaphysical?) metavocabulary quite distinct from the base vocabulary for which a semantics is to be provided.

vi) Various different *reason relations* among sentences can then be defined, appealing either *just* to the mereological structure of the universe and semantic space (as Fine does for consequence and incompatibility—which will appeal to false-makers), or also to the *modal* structure, as Ulf's definition of consequence does (and a stronger notion of incompatibility that appealed to *impossibility* would).

In *both* cases, given a semantics for a base vocabulary, semantic interpretants can be assigned to sentences (and sets of sentences) in the *logically extended* lexicon that can be elaborated from it. The *interpretation function* for sentences from the logically extended lexicon is wholly determined by the interpretation function for sentences in the base lexicon.
Reason relations can then be computed for those sentences, to reconstruct various familiar *logical* consequence and incompatibility (inconsistency) relations. No further **stipulation** is required in either semantic framework.

Slogan: **Implication-space conceptual role semantics is *semantics without metaphysics*.**
In addition, it does without *semantic stipulation* of an interpretation function that associates elements of the *universe*, specified in the *metaphysical* vocabulary, with *sentences* as their *semantic interpretants*.
In pure conceptual role semantics, all this semantic structure is elaborated directly from the reason relations of the vocabulary for which a semantics is being supplied.
This is what I mean by saying that **the implication-space conceptual role semantics is the *intrinsic* semantics of reason relations as such**.
It is *intrinsically universal*, elaborated from *any* reason relations, hence from *any* vocabulary.

Now one might just say: "But I *like* metaphysics."
That expression of mere preference would perhaps be backed up with good reasons.

For a natural thought is that in doing without the metaphysics—or leaving it out, as it might appear—one is abstracting away from something semantically essential: relations to the *world* in which discursive practices are conducted.

That thought deserves to be taken seriously—and bimodal hylomorphic conceptual realism is one way of doing so. (Though we will need to fill it in with they story about how what is expressed in alethic and deontic modal vocabularies—the "modalities" in "bimodal"—are related by two sorts of relations: epistemic tracking of representeds by representings, expressed in alethic modal vocabulary, and semantic governance of representings by representeds, expressed in deontic normative vocabulary. [A topic for later meetings.])

But it also comes with a substantial task (of the sort Lewis acknowledges in "Languages and Language"). One must specify how sentential expressions must be *used* so as to institute the relations between semantic interpretants and sentences that is at the core of metaphysical semantics. One must say what practitioners *do* that confers on them the semantic content that the semantic interpretation function assigns to them. The activities, practices, and attitudes that establish the association of semantic interpretants with items of the lexicon, which makes the difference between a *lexicon* and a *vocabulary*, according to metaphysical semantics, must be specified in some *pragmatic* metavocabulary.

Doing that **is doing *philosophical* semantics, not just *formal* semantics**. Formal semantics can proceed on the basis of mere stipulation of the relations between expressions and semantic interpretants. It is of the essence of philosophical semantics that it cannot—that it shoulders the explanatory burden of explaining the association between practical acts and attitudes, on the one hand, and semantic interpretation functions relating semantic interpretants specified in a metaphysical vocabulary and items in the lexicon of the base vocabulary whose semantics is being articulated in the semantic metavocabulary.

And we take on that explanatory-explicative responsibility and fulfill it. For we *have* an account, couched in the terms of a bilateral, deontic normative pragmatic metavocabulary, of what it is to *use* expressions so as to confer on them roles with respect to reason relations of implication and incompatibility.

That is to say what it is to *use* the items of a lexicon so as to institute reason relations among them and so constitute them *as* a *vocabulary*. And we can do that for *every* vocabulary, just insofar as it *is* a vocabulary. That, we claim, is a signal advantage of deploying implication-space conceptual role semantics, just because it is *intrinsic* to the reason relations that articulate the semantic interpretants it associates with lexical items. At the end of the next section, I will say something about how the mapping between implication-space conceptual role semantics and truthmaker semantics holds some promise of applying and extending our story to begin to remedy the lack of a pragmatics in the truthmaker setting.

VI. Implication-Space Conceptual Role Semantics and Truthmaker Semantics Share a Structure and Can Express the Same Reason Relations

In spite of the substantial difference in the conceptions of semantic interpretation that animate the two different approaches, the fact that both take the mathematical form of commutative monoids plus distinguished subspaces means that their treatment of the crucial reason relations of implication and incompatibility share enough structure to be intertranslatable across the two settings. That is, **we can specify exactly the same reason relations of implication and incompatibility while moving systematically between the modalized state spaces of truth-maker semantics and implicational phase-space semantics**. Here's how. [This technical result is my own small contribution to the enterprise that Dan and Ulf have given shape to with their major constructions and results.
I'm not going to rehearse these technical details. I record them on the handout for the record and in case anyone is curious about them. What matters is the results, summarized in bold below.]

For one direction: Beginning with a truth-maker model, one can define an implicational phase space that corresponds to it in the sense of defining exactly the same implications and incompatibilities. We are given a truth-maker model of a language L_0, defined on a modalized state space $<S,S^\Diamond,\sqcup>$, which assigns to each sentence $A\in L_0$ a pair of sets of states $<v(A),f(A)>$ understood as verifiers and falsifiers of that sentence. The points of the implicational phase space being defined are ordered pairs of sets of sentences of L_0. These are the candidate implications. What corresponds to fusion, $\sqcup$, is adjunction: $<\Gamma,\Delta> \uplus <\Theta,\Psi> = <\Gamma\cup\Theta,\Delta\cup\Psi>$, as usually defined in implicational phase space semantics. It remains to compute $\mathbf{I}_0$, the set of *good* implications. We do that using the consequence relation Hlobil defined to mimic the Restall-Ripley bilateral understanding of the multisuccedent turnstile:

$$<\Gamma,\Delta>\in\mathbf{I}_0 \quad \text{iff} \quad \forall s,t\in S[\ (\forall G\in\Gamma[s\in v(G)]\ \&\ \forall D\in\Delta\ [t\in f(D)]\ ) \Rightarrow s\sqcup t\notin S^\Diamond\ ].$$

That is, $<\Gamma,\Delta>$ is a good implication just in case the fusion of any state s that verifies all of Γ and any state t that falsifies all of Δ is an impossible state, in the truth-maker model. This construction obviously guarantees that exactly the same implications will hold in the implicational phase space, that is, be elements of $\mathbf{I}_0$, as satisfy the Hlobil consequence relation in the truth-maker model.

As for incompatibilities, in the truth-maker setting, two *states* s and t are incompatible just in case their fusion is an impossible state. Two *sentences* A and B are incompatible just in case any fusion of a verifier of the one with a verifier of the other is an impossible state. More generally, a set Γ of sentences is *incoherent* in case any fusion of verifiers of all its elements is an *impossible* state. Given the definition of the set of good implications $\mathbf{I}_0$ just offered, this is equivalent to $<\Gamma,\varnothing>\in\mathbf{I}_0$. The incompatibilities are represented in the implicational phase space semantics just by good implications with empty right-hand sides.

So there is a straightforward method for taking any truth-maker model defined on a modalized state space and defining from it an implicational phase space model that has exactly the same reason relations of implication and incompatibility.

For the other direction: **Beginning with an implicational phase space, one can define a truth-maker model (an interpreted modalized state space) that corresponds to it in the sense of defining exactly the same implications and incompatibilities**.
We are given an implicational phase space defined on a language L_0, $<\mathcal{P}(L_0) \text{ x } \mathcal{P}(L_0), \mathbf{I}_0>$. The states will be candidate implications. $S = \mathcal{P}(L_0) \text{ x } \mathcal{P}(L_0)$. $\sqcup$ is adjunction: $<\Gamma,\Delta> \sqcup <\Theta,\Psi> = <\Gamma\cup\Theta,\Delta\cup\Psi>$. In the Hlobil truth-maker definition of consequence, the *good* implications correspond to *im*possible states. So the subset of *possible* states is defined by $S^{\Diamond} = S\text{-}\mathbf{I}_0$. It remains to define the model function m, which assigns to each $A\in L_0$ a pair of subsets of S, $<v(A),f(A)>$, where $v(A)\subseteq L_0$ and $f(A)\subseteq L_0$, such that:

$$<\Gamma,\Delta>\in\mathbf{I}_0 \text{ iff } \forall s,t\in S[(\Gamma=\{G_1\ldots G_n\} \;\&\; g_1\in v(G_1) \;\&\ldots g_n\in v(G_n) \;\&\; s=g_1\sqcup\ldots\sqcup g_n \;\&\; \Delta=\{D_1\ldots D_n\} \;\&\; d_1\in v(D_1) \;\&\ldots d_n\in v(D_n) \;\&\; t=d_1\sqcup\ldots\sqcup d_n) \Rightarrow s\sqcup t\notin S^{\Diamond}].$$

For various metatheoretic purposes, Fine employs “canonical” truth-making models, in which the verifier of a (logically atomic) sentence is just that sentence and the falsifier of that sentence is just the negation of that sentence. (His requirement that the fusion of any verifiers of A will be a verifier of A and the fusion of any falsifiers of A will also be a falsifier of A is then trivially satisfied, since there is only one.) We can combine that idea with Kaplan’s standard representation of the proposition expressed by A as the pair $< <A, \varnothing>, <\varnothing, A> >$, and do without the formation of falsifying literals by appeal to negation by defining the verifiers of A by $v(A) = <A, \varnothing>$ and the falsifiers of A by $f(A) = <\varnothing, A>$. We want to implement Hlobil’s definition of implication (generalizing C. I. Lewis’s strict implication to Fine’s truthmaker semantic framework), that an implication $\Gamma|\!\sim\Delta$ is good in the truth-maker setting just in case the fusion of any verifier of all of Γ and any falsifier of all of Δ is an impossible state. To do that, we need to say what it is for a state (defined in the implicational phase space, that is, a candidate implication) to “verify all of Γ” and to “falsify all of Δ.” We can extend the single-sentence definitions as follows. If $\Gamma=\{G_1\ldots G_n\}$ and $\Delta=\{D_1\ldots D_m\}$:

$$v(\Gamma) = <\Gamma,\varnothing> = <G_1,\varnothing>\uplus\ldots\uplus <G_n,\varnothing>.$$

$$f(\Delta) = <\varnothing,\Delta> = <\varnothing,D_1>\uplus\ldots\uplus <\varnothing,D_m>.$$

That is, the implication (standing in for a state) $<\Gamma,\varnothing>$ counts as verifying all of Γ because it is the adjunction of the verifiers of each element of Γ. (In this “canonical” modalized state-space model, sets of sentences, like individual sentences, only have single states=implications as verifiers.) And similarly for falsifiers.

To show that this works, in the sense of yielding the same implications in the truth-maker model that are good in the original implicational phase space, we must show that

$$\langle\Gamma,\Delta\rangle\in\mathbf{I}_0 \text{ iff } \forall s,t\in S[\ (\forall G\in\Gamma[s\in v(G)]\ \&\ \forall D\in\Delta\ [t\in f(D)]\ ) \Rightarrow s\sqcup t\notin S^{\Diamond}\ ].$$

[This is not quite right. It is sufficient, I think, but not necessary.
For the *semantic entailment* relation that is the real reason relation corresponding to implication in the implication-space setting is defined using *v*-function closures of sentences. And this is necessary for it to extend to logically complex sentences. Further, even implications that are not in $\mathbf{I_0}$ can be good, if they involve sentences implicationally equivalent to (playing the same role in good implications as) sentences that *do* show up in $\mathbf{I_0}$. (Is this right, *can* they "play the same role w/res to good implications if they do *not* show up in $\mathbf{I_0}$?)
Semantic Entailment in implication space is:

$$A \models_M B \text{ iff } (([A]_P)^{v} \dot{\cup} ([B]_C)^{v})^{vv} \subseteq \mathbf{I}_M.$$

This is what I should really be using here.
The rationale for this shortcut is that *if* $\langle A,B\rangle\in\mathbf{I_M}$, *then* $A \models_M B$.]

To show the left-to-right direction $\Rightarrow$: If $\langle\Gamma,\Delta\rangle\in\mathbf{I}_0$ then $v(\Gamma)=\langle\Gamma,\varnothing\rangle$ and $f(\Delta)=\langle\varnothing,\Delta\rangle$. So $v(\Gamma)\sqcup f(\Delta)=\langle\Gamma,\Delta\rangle$. Since by hypothesis $\langle\Gamma,\Delta\rangle\in\mathbf{I}_0$, by the definition of $S^{\Diamond}$ as $S\text{-}\mathbf{I}_0$, it follows that $\langle\Gamma,\Delta\rangle\notin S^{\Diamond}$, that is, that the state $\langle\Gamma,\Delta\rangle$ is an impossible state. It is the fusion of *the* verifier of Γ, $\langle\Gamma,\varnothing\rangle$ and *the* falsifier of Δ $\langle\varnothing,\Delta\rangle$ because it is the result of adjoining them.
To show the right-to-left direction $\Leftarrow$: If $\forall s,t\in S[(\Gamma=\{G_1\ldots G_n\}\ \&\ g_1\in v(G_1)\ \&\ldots g_n\in v(G_n)\ \&\ s=g_1\sqcup\ldots\sqcup g_n\ \&\ \Delta=\{D_1\ldots D_n\}\ \&\ d_1\in v(D_1)\ \&\ldots d_n\in v(D_n)\ \&\ t=d_1\sqcup\ldots\sqcup d_n\ ) \Rightarrow s\sqcup t\notin S^{\Diamond}\ ]$, then $s = v(\Gamma)$ and $t = f(\Delta)$, so $v(\Gamma)\sqcup f(\Delta)=\langle\Gamma,\Delta\rangle \notin S^{\Diamond}$. Since $S^{\Diamond} = S\text{-}\mathbf{I}_0$ and $\langle\Gamma,\Delta\rangle\in S$, $\langle\Gamma,\Delta\rangle\in\mathbf{I}_0$.

As for incompatibility, we must show that A and B are truth-maker incompatible ($\Gamma$is truth-maker incoherent), that is, $\forall s,t\in S[s\in v(A)\ \&\ t\in v(B) \Rightarrow s\sqcup t\notin S^{\Diamond}]$, (or more generally, $v(\Gamma)\notin S^{\Diamond}$) iff $\langle\{A,B\},\varnothing\rangle\in\mathbf{I}_0$ (or more generally, $\langle\Gamma,\varnothing\rangle\in\mathbf{I}_0$).
To show the left-to-right direction $\Rightarrow$: If $\forall s,t\in S[s\in v(A)\ \&\ t\in v(B) \Rightarrow s\sqcup t\notin S^{\Diamond}]$, then since $v(A) = \langle A,\varnothing\rangle$ and $v(B) = \langle B,\varnothing\rangle$, and since $\sqcup$ is adjunction, $s\sqcup t = \langle\{A\}\cup\{B\},\varnothing\rangle = \langle\{A,B\},\varnothing\rangle$. Since $\Rightarrow s\sqcup t\notin S^{\Diamond}$, $s\sqcup t = \langle\{A,B\},\varnothing\rangle\in\mathbf{I}_0$. This works for arbitrary iterations of $\sqcup$, which gives the more general Γcase.
To show the right-to-left direction $\Leftarrow$: If $\langle\{A,B\},\varnothing\rangle\in\mathbf{I}_0$, then $\langle\{A\}\cup\{B\},\varnothing\rangle\in\mathbf{I}_0$.
Since $\sqcup$ is adjunction, $\langle A,\varnothing\rangle\sqcup\langle B,\varnothing\rangle\in\mathbf{I}_0$. But $v(A) = \langle A,\varnothing\rangle$ and $v(B) =\langle B,\varnothing\rangle$.
So $v(A)\sqcup v(B)\in\mathbf{I}_0$. Since $S^{\Diamond} = S\text{-}\mathbf{I}_0$, $v(A)\sqcup v(B)\notin S^{\Diamond}$. That is truth-maker incompatibility of A and B. This works for arbitrary iterations of $\sqcup$, which gives the more general Γ case.
End of Demonstration.

Let me try to be careful in characterizing what this argument purports to show.
It is that *if* reason relations of implication and incompatibility are defined in the truthmaker framework in a particular way—what we claim is the *right* way—then exactly the same relations

of implication and incompatibility are specifiable in the implication-space conceptual role framework, and *vice versa*.

An implication Γ |~ Δ is taken to hold in the truthmaker semantics just in case the fusion of any truthmakers of all of Γ with falsemakers of all of Δ is an impossible state.
An incompatibility Γ#Δ is taken to hold in the truthmaker semantics just in case the fusion of any truthmakers of all of Γ with truthmakers of all of Δ is an impossible state.

And it should be acknowledged that these are *not* the definitions of implication and incompatibility that Fine himself proposes and endorses.

By contrast to the definitions of reason relations Fine uses, these are both essentially *modal* definitions. Fine himself does not appeal to the distinction between possible and impossible states in any of the definitions of consequence he offers.
(His Entailment looks only at set-theoretic inclusion relations among truthmakers, and his Containment adds mereological structure, but not modal structure. He doesn't explicitly define incompatibility, but his definition of negation, which codifies material incompatibilities, is entirely in terms of truthmakers and falsemakers.)
We can construct analogues of sentential truthmakers and falsemakers in terms of implication-space conceptual roles.
They correspond to premissory and conclusory roles.

So presumably we could also define analogues of the *nonmodal* definitions of implication and incompatibility that Fine actually uses. I think it is pretty clear how that would go with his notion of Entailment, where A |~ B iff every truthmaker of A is a truthmaker of B. The mapping I have defined shows us how to mirror that relation in set-theoretic inclusion relations among the *v*-sets of sentences (and sets of sentences). It also offers a way of mirroring the mereological part-whole relations among truthmakers that Fine appeals to in defining consequence as what he calls "Containment." The key concept there is that every truthmaker of one proposition is part of a truthmaker of the other, and every truthmaker of the second has a truthmaker of the first as one of its parts. The mapping I offered above shows that and how we can *translate* this into the metavocabulary of implication spaces. But work would need to be done to show that the result behaved suitably *as* a consequence relation in the sparer implication-space setting.

In any case, I hope the strategy is clear. I have set up an isomorphism at the level of what in Belnap's terms (about which I offered some reservations) is the *presemantics* (determining the universe from which semantic interpretants are to be constructed) and the *semantics* (defining a function that assigns semantic interpretants to sentences in the lexicon). That provides all the raw materials for a translation back and forth between the truthmaker and implication-space settings of what Belnap calls the *postsemantics*: the definition of the reason relations among the sentences in terms of the association with them of semantic interpretants. Reason relations that are definable in the postsemantics of either semantic setting can be mapped onto ones definable

in the other. The question of whether they behave like reason relations when so translated must be investigated in each particular case.
[Explore this suggestion further. The role of *v*-sets definitely introduces complications.
For we need an **I** set to get sentential roles that correspond to truthmakers (and falsemakers), since the analogue of his Entailment will look at set-theoretic inclusion relations among *v*-sets of premissory roles: <A,∅>.
If we use (the converse of) Fine's $S^{\Diamond}$, then changing that will change the consequence relation.
But this consequence relation of Fine's is not modalized, and so is independent of $S^{\Diamond}$.
So we need to use his Entailment consequence relation to determine **I**, and so the *v*-sets.
And it might be difficult to formulate structural constraints that would make Fine's definitions work in this setting. One would certainly not expect the result to be robust in the full range of open-structured, nonmonotonic, nontransitive values of **I** for which we can show Ulf's definition of consequence *is* robust.]

So I have confined myself here to showing how the kinds of reason relations that we think are most important can be expressed equivalently in the metaphysical truthmaker semantic metavocabulary and in the implication-space conceptual role metavocabulary.
One important reason to think that these modalized versions *are* the most important notions of implication and incompatibility is, of course, that we know how to connect them to the *use* of vocabularies exhibiting those reason relations, as specified in deontic normative *pragmatic* metavocabularies. That is just what Ulf's isomorphism shows. We can tell a *philosophical* semantic story about these reason relations and the conceptual roles they articulate, not just a *formal* semantic story about them. One possibility, which should be investigated, is that the isomorphism I have constructed here can be exploited to supply what the truthmaker semantic framework as it stands does not have: a pragmatic story about how semantic interpretants as it understands them are associated with the sentences it assigns them to by the *use* of those declarative sentences in discursive practices of making claims and rationally challenging and defending them. If and insofar as that *can* be done, the benefits would run in both directions. For if that story could be run in the *other* direction, it would show how to connect the implication-space semantics to a metaphysical account of the worldly states that serve as truthmakers and falsemakers.

I conclude this section with a summarizing question:
Why is the result I have presented here important? What does it show?
Ulf shows an isomorphism between the reason relations defined by truthmaker semantics and those defined by Restall-Ripley normative pragmatics.
Dan showed that implication-space semantics is sound and complete for the logic NM-MS.
Showing that the implication-space semantics is a meta-metavocabulary for the reason relations of truthmaker semantics (subject to the qualifications I just outlined) accordingly shows that it

characterizes the reason relations of all three corners of the triangle of rational metavocabularies with which we began.
This is what justifies elevating it to the apex of the triangular pyramid, as articulating the intrinsic structure of reason relations as such—the topic addressed, each in its own way, by pragmatic, logical, and semantic *rational* metavocabularies.

Here I cannot resist inserting a teaser.

Later on in the seminar I will report on another direction in which we have explored the relations between reason relations of the form I here show to be common to the truthmaker and implication-space semantics, on the one hand, and normative pragmatics, on the other.

Another member of our ROLE logic group, Pitt philosophy Ph.D. student Yao Fan, has, under my supervision, written a computer program in Python ("**Dialogic Pragmatics 1**" or DP1) that, when given as input only a *vocabulary* in the technical sense I use the term in here—a lexicon and a set of reason relations on that lexicon, represented as a set of pairs of sets of sentences from the lexicon—produces dialogical exchanges between interlocutors, who *make claims* and *rationally challenge* and *defend* those claims by offering *reasons for* and *against* them.

The aim is to show that the definition of reason relations in a deontic normative pragmatic metavocabulary can be exploited in the *other* direction, to move from specifications of reason relations to actual norm-governed discursive practices.

VII. Comparing Truth-Based Semantics and Conceptual Role Semantics: The example of Multivalued Logics and Inferential Entailment Roles.

1. Intersubstitutability w/res to just premises, or just conclusions, vs. both. Assuming that these line up is inconsequentiality of explicitation. It is assuming sentences "mean the same" on both sides of the turnstile.

Contra propositions, in favor of premissory and conclusory roles.
Talk of claimables as *propositions* assumes that internal logic and external logic are the same, that sentences "play the same role on both sides of the turnstile."
The thought is that if they don't, then they are *ambiguous*, and we are equivocating and not reasoning properly.
But that is just a structural *closure* prejudice.
In fact, it is the assumption that *explicitation is inconsequential*, and so that *inference*—*drawing conclusions*, in the sense of actually accepting or rejecting what one is *implicitly* committed to accept or reject, based on one's other commitments, never makes a practical difference.
As soon as we aspire to deal (logically and semantically) with *open* structured (substructural) reason relations, we need to cut things finer than "proposition" talk does.

In general, we want to consider what we can learn from the *relations* among what we can say about reason relations (and so, about discursive practice) in various kinds of metavocabularies: pragmatic, semantic, logical, and the native implicational semantic.
One aspect of this is learning about the relations between inference-based and truth-based semantic approaches.

Here I want to consider the material that Dan presented in the second half of his talk last time: the discussion of *Inferential Entailment Roles*.
As he and I discussed then, one way to think about what he is doing is moving from thinking of conceptual roles in terms of noting substitutional invariances *salve veritate* (preserving truth), as the tradition does, to thinking of conceptual roles in terms of noting substitutional invariances *salve consequentiae* (preserving consequences), as we do.
That is, instead of observing which substitutions of sentences preserve *truth-value*, we look at which ones preserve the *goodness* of *implications* (and incompatibilities).

The traditional truth-based semantic framework it is most useful to consider here is not the sophisticated and expressively flexible truthmaker semantics, but the old-fashioned (but newly important) three-valued (and four-valued) semantics for sentential logical vocabulary.

They go back to Peirce, flourished in the middle years of the twentieth century, were given additional impetus by Kripke's use of them to address semantic paradoxes, and have lately had a renaissance of interest and development (only some of which I will so much as mention).

We can begin with the observation that the *point* of talk of truth *values* in formalized semantics *always* was to understand consequence (implication), and, less explicitly, incompatibility-incoherence-inconsistency.

The basic idea is the traditional one, exploited by Frege, that that **good implications never take us from true premises to false conclusions**. That is the thought that good implications *preserve truth*.
We have seen a sophisticated *pragmatic* elaboration of that idea in Restall and Ripley's bilateral normative pragmatic understanding of implication.
It is that good implications normatively rule out asserting all the premises and denying all the conclusions.
Asserting expresses practical acceptance, which is taking-*true*.
Denying expresses practical rejection, which is taking-*false*.
So they are defining what it is practically to take an inference to be *good* as acknowledging a prohibition against taking-*true* the premises and taking-*false* the conclusions.
That later pragmatic characterization turns out to be *much* more flexible than the truth-functional logical implementation of the first.
That idea (that good implications do not have true premises and conclusions that are not true) is what is generalized to multivalued logics.
There it takes the form of the principle that good implications are those where *designatedness* of value is preserved.

Consider the following generalizations of the classical two-valued truth-functional definition of logical connectives:

¬	**A**	**¬A**
	T = 1	F = 0
	U = ½	U = ½
	F = 0	T = 1

A&B B: **A**	T = 1	U = ½	F = 0
T = 1	T = 1	U = ½	F = 0
U = ½	U = ½	U = ½	F = 0
F = 0	F = 0	F = 0	F = 0

A∨B B: A	T = 1	U = ½	F = 0
T = 1	T = 1	T = 1	T = 1
U = ½	T = 1	U = ½	U = ½
F = 0	T = 1	U = ½	F = 0

Where all the atomic sentences are either true or false, these are just the truth-tables for classical logic. But when one of the components is the third value, so is its negation, and so is its conjunction with anything that is not false, and so is its disjunction with anything that is not true.

Have we yet specified, in semantic terms, a *logic*?
The universe is three truth-values. (That is Belnap's "presemantics".)
The tables tell us how to assign elements of that universe to sentences, recursively, as semantic interpretants. (That is Belnap's "semantics".)
But we have not yet offered a definition of *reason relations*, in particular *implication*, in terms of those assignments of semantic interpretants.
This is Belnap's "post-semantics".
But recall that I complained about this aspect of his terminology.
You have not offered a semantics until you have defined *reason relations* in terms of your assignment of semantic interpretants.
(The definition of negation does not yet even define incompatibility.
For that we will need to add that A#B iff for some X, A|~X and B|~¬X.
And we have not yet defined implication.)

To define implication, we are to be guided by Frege's principle that a good implication never has true premises and false conclusions.
But there are different ways of extending that principle to the three-valued semantic universe.
We could replace 'false' with 'not true', and exclude candidate implications with true premises and conclusions of either of the other two truth-values.
Or we could replace 'true' with 'not false', and exclude candidate implications with false conclusions and premises of either of the other two truth-values.

The first option gives us the logic K3 (Weak Kleene):

A|~B B: A	T = 1	U = ½	F = 0
T = 1 *	✓	X	X
U = ½	✓	✓	✓
F = 0	✓	✓	✓

The second option gives us the logic LP (Graham Priest's "Logic of Paradox"):

A|~B B: **A**	T = 1	U = ½	F = 0
T = 1 *	✓	✓	X
U = ½ *	✓	✓	X
F = 0	✓	✓	✓

These are *different* logics, even though their semantic definitions of the connectives are the *same*, in the sense that they assign exactly the same elements of the universe (of three truth-values) as semantic interpretants to all the sentences.
The difference lies *only* in the definition of implication (and therefore, given negation, of incompatibility).

Frege's principle for determining good implications from truth-values can be reformulated in the 3-valued setting as the principle that:
Good implications never have premises assigned *designated* truth-values and conclusions assigned *undesignated* truth-values.
In the tables above, I have marked the designated values with an asterisk '*'.
K3 is what you get if you treat *true* as the only designated value.
LP is what you get if you treat not only *true*, but also the intermediate value, ½, as designated, in both cases, for the purposes of defining "good implication" from the assignment of truth-values as semantic interpretants.

Now, from the philosophical point of logical expressivism, these formally well-behaved semantics are barely intelligible as conferring meaning on *logical* vocabulary. Neither of the conditionals codify the implication relations that define K3 and LP (post)semantically. Detachment (*modus ponens*) does not hold for either of these conditionals. And it is a challenge to say what sense of "incompatibility" it is that their negations capture. But these complaints about the *point* of these logics aren't relevant to the lessons I want us to learn from them.

For the determination of reason relations on this neo-Fregean model, what matters is not which *multivalue* a sentence is assigned, but only whether or not it is a *designated* multivalue.
For good implications are all and only those that *preserve designatedness*.
Q: So what difference *does* the difference between different multivalues with the same designatedness value (designated/undesignated) make?
A: Multivalues codify the contribution sentences make to the designatedness of the multivalue semantically assigned as *components* of *compound* sentences containing them.

This fact is the key to understanding the difference between employing this semantic apparatus *synthetically*, to compute which implications are good, and employing it *analytically*, to sort sentences into semantic equivalence classes—as playing the same *conceptual role*—based on an antecedent sorting of implications into good and bad ones.
That distinction of implications into good and bad ones is just what a *vocabulary* does, in the technical sense in which I have been using that term.
When this semantic apparatus is used *analytically*, two sentences are assigned the same *multivalue* (here, 1, 0, or ½: True, False, or Other) just in case substituting one for the other as components of compound sentences never turns a good implication into a bad one.
And two multivalues are assigned the same *designatedness* value just in case substituting one for the other as the semantic interpretant of a free-standing sentence appearing as premise or conclusion of an implication never turns a good implication into a bad one.

Side note: If one starts with a *logical* consequence relation (in this usage, bringing with it a specification of incompatibility relations), defined over a lexicon, then using this *analytic* method of assimilating sentences into equivalence classes by noting substitutional invariances *salva consequentiae*, and treating *theoremhood* in the logic as designatedness, yields a *Lindenbaum algebra* for that logic. In that algebra, the equivalence classes of sentences count as multivalues that yield a sound and complete multivalued (matrix-valued) semantics for that logic.
I discuss the significance of this construction in the first part of Chapter Seven of *Making It Explicit*.

This analytic employment of semantic apparatus of multivalues and designatedness is important to keep in mind. For it is when we think of it that way that we can best see the claim I am shaping up to argue for.
That is that Dan's Inferential Entailment Roles (IER) metavocabulary for specifying conceptual roles in the context of implication-space semantics is *much* more expressively powerful than the multivalued truth-value semantic metavocabulary for doing so.

Here is the short version of the argument for that claim.
Put in terms of the IER metavocabulary for conceptual roles, it turns out (Dan has discovered) that K3 is the logic of pure *premissory* roles (the roles sentences play as premises of implications) and LP is the logic of pure *conclusory* roles (the roles sentences play as conclusions of implications).

Here is how to think about premissory and conclusory roles.
Think first of a fully classical setting in which implication relations have a closure structure, that is are transitive and monotonic. In such a setting we can say all three of the following things.

1) 'Pedro is a donkey' *implies* 'Pedro is a mammal': D(P) |~ M(P).
2) Anything that follows from (is implied by) 'Pedro is a mammal' follows from 'Pedro is a donkey':

If Γ,M(P) |~ A, then Γ,D(P) |~ A.

That is, 'Pedro is a donkey' is implicationally *stronger* as a premise, in the sense that it *implies more* than 'Pedro is a mammal.'

3) Anything that *implies* ‘Pedro is a donkey’ *implies* ‘Pedro is a mammal’:

If Γ|~D(P), then Γ |~ M(P).

That is, ‘Pedro is a mammal’ is *stronger* as a conclusion than ‘Pedro is a donkey’ in the sense that is *implied by more*.

In the fully structural setting, all of these go together.

If implication has an open structure, by contrast, these three can come apart.

The home language-game of talk of sentences as “expressing propositions” is settings such as this, in which sentences “mean the same thing on both sides of the turnstile,” in precisely the sense that what is often called their “internal” logic, what implies what, coincides with their *external* logics in the sense of both premissory and conclusory role entailments.

Note: The recently introduced and intensively studied three-valued logic ST (for Strict/Tolerant) is more expressively powerful than K3 or LP, and as making possible the codification of *nontransitive* reason relations, in virtue of its abandonment of the Fregean semantic model of goodness of implication as corresponding to the *preservation* of something. For the *preservation* model builds in transitivity.

One of the measures of the substantially greater expressive power of Dan’s Inferential Entailment metavocabulary for characterizing conceptual roles is that *none* of these multivalued logics can handle *nonmonotonic* implication relations—whereas his can handle reason relations that are nonmonotonic, nontransitive, or both.

The metavocabulary of Inferential Entailment Roles can specify premissory and conclusory roles.

If substituting A for B as a premise never turns a good implication into a bad one, then Dan will write $A^P \Rightarrow B^P$.

If substituting A for B as a conclusion never turns a good implication into a bad one, then Dan will write $A^C \Rightarrow B^C$.

But he defines *inferential entailment roles* semantically in the implication-space metavocabulary, in a *much* more general fashion.

$$A^P,B^C \Rightarrow C^P,D^C$$

$$\text{iff}$$

$$[A]_P \cap [B]_C \subseteq (([C]_P)^\nu \sqcup ([D]_C)^\nu)^\nu.$$

This formulation codifies the case where whenever A occurs as a premise and B as a conclusion, substituting C for A as a premise and D for B as a conclusion will never turn a good implication into a bad one.

This *mixes* premissory and conclusory roles, and specifies a much more complex relation among sentences than simple premissory and conclusory roles does.

It accordingly provides a much finer expressive scalpel for exposing and dissecting the fine structure of relations among the conceptual roles played by sentences.

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Appendix on K3 and LP:

Here are a few more facts about K3 and LP, as they show up in the truth-value semantic setting. Our claim is that looking at these facts from the expressively richer perspective afforded by implication-space and inferential entailment metavocabularies for specifying and manipulating conceptual roles sheds new light on all these phenomena.

- K3 and LP are *duals*, in that

a) $\Gamma \models_{\mathbf{K3}} \Delta$ **iff** $\neg\Delta \models_{\mathbf{LP}} \neg\Gamma$ and

b) $\Gamma \models_{\mathbf{LP}} \Delta$ **iff** $\neg\Delta \models_{\mathbf{K3}} \neg\Gamma$

(where '$\neg\Gamma$' indicates the set consisting of the negations of all the elements of Γ).
Because in multisuccedent sequent calculi negation is defined by moving sentence across the turnstile, this duality is an immediate consequence of understanding:
K3, as the logic of premissory metainferences, which requires reading sequent-calculus proof trees top to bottom and
LP, as the logic of conclusory metainferences, which requires reading sequent-calculus proof trees bottom to top.

- K3 and LP interpret the third truth-value in complementary ways.

c) K3 in effect reads its third truth value as *neither* true *nor* false.

d) LP in effect reads its third value as *both* true *and* false.

Belnap's tetralogic and the corresponding bilattice that Dan talked about last week include both of these additional "truth values." K3 and LP just extract different three-valued logics from that four-valued bilattice.
Here we can think of the universe of the semantics as the *set* {0,1} (or {True, False}) and the possible values that can be assigned to sentences as semantic interpretants as *subsets* of that universe. There are four of them: {1}, {0}, {1,0}, and {} (the empty set)—True, False, Both, and Neither.
K3 uses {1}, {0} and {}, and
LP uses {1}, {0}, and {1,0}.
Both can then be understood as treating a value as designated iff it contains 1 as an element.
And both can then be understood as treating an implication as good iff it does not have all designated premises and no designated conclusion.

- K3 and LP are logics of truth-value *gaps* and *gluts* respectively. Specifically:

e) The result of K3 treating the third value as {}, neither true nor false (c), is that it does not have the Law of Excluded Middle (LEM) as a theorem.

It would if for arbitrary assignments of semantic values to the elements of any premise-set **Γ|~A,¬A**. But it is easy to check that this will *not* be a valid implication for K3 if all of Γ is assigned value 1 and A is assigned ½.

f) The result of LP treating the third value as {1,0}, both true and false (d), is that it does not have the Law of NonContradiction (LNC) in the form of *explosion*: that *everything* follows from a contradiction.

It would if for arbitrary assignments of semantic values to Γ, A, and B, **Γ,A,¬A|~B** held. But this will not be a valid LP implication if all of Γ is assigned value 1, A is assigned value ½, in which case ¬A will also get ½, and all the premises will be designated, but B can be assigned value 0, so that the implication fails.
Logics without LEM, such as K3, are called *paracomplete*.
Logics without LCN, such as LP,, are called *paraconsistent*.

- K3 and LP can be and have been used to block semantic paradoxes.

The argument that Liar sentences blow up consequence relations in vocabularies that include transparent truth predicates standardly goes something like this:
If A is a liar sentence ('A' is false), then If A is true, then A is false, and if A is false, then A is true: A |~ ¬A and ¬A |~ A. But A must be either true or false (LEM). So A is true and A is false: A, ¬A. but A,¬A |~ B, for arbitrary B (LNC). K3 blocks one step, LP another.
Put another way, K3 can treat the Liar as neither true nor false, and LP can treat it as both.

Understanding K3 as the logic of premissory metainferences (reading *down* proof trees in NM-MS) and LP as the logic of conclusory metainferences (reading *up* proof trees in NM-MS, which uses Ketonen's reversible rules) explains why K3 shows up as *gappy* and paracomplete and why LP shows up as *glutty* and paraconsistent.

The short story, as Ulf explains it, is that NM-MS does not have Cut (CT), transitivity, across the turnstile (in the "internal logic"). That has as a consequence the *undeniability* of A&¬A on the right of the turnstile (no other denial logically entitles you to deny this contradiction), as conclusion and the *unassertibility* of A∨¬A on the left, as a premise (no other assertion logically entitles you to assert this). That is what shows up as gluts on the conclusory side, in LP, and as gaps on the premissory side, in K3.